Constructing Observable Bounds for Productivity Index

Hideyuki Mizobuchi and Valentin Zelenyuk

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1. Motivation

2. Model

3. Results

4. Empirical application

5. Conclusion

1. Motivation

Index number problem

- Empirical index (index number formula) is formula of prices and quantities at two comparison periods
- Numerous empirical indices have been proposed before
- How to select one of them? (Index number problem)
- Various approaches to this problem exist
- This paper focuses on economic approach

Economic approach to index number theory

- Propose a theoretical index by using economic theory
  - The theoretical index is regarded as the true index

- Evaluate empirical index by relating with the theoretical index
  - Empirical index is close to theoretical index $\Rightarrow$ Good index
  - Empirical index is far from theoretical index $\Rightarrow$ Bad index

- Two distinct strands within this approach

Two strands in economic approach

- Strand for pursuing exactness to theoretical index
  - Optimizing behavior + Functional form ⇒ Justify a single index
  - Konus & Byushgens (1926), Diewert (1976), Caves, Christensen & Diewert (1981), Diewert (1992)

- Strand for pursuing reasonable bound
  - Optimization ⇒ Justify any index within a observable bound
  - Konüs (1924), Pollak (1990), Färe & Grosskopf (1992), Balk (1993), Balk (1998)

Case of consumer price index

- Cost of living index (COLI) is a representative theoretical index for consumer price
  - COLI is defined on the reference utility level

- Konüs (1924)
  - Laspeyres index $\geq$ COLI based on period 0 (=Laspeyres COLI)
  - Paasche index $\leq$ COLI based on period 1 (=Paasche COLI)

- Pollak (1990)
  - Under homotheticity, Paasche index $\leq$ COLI $\leq$ Laspeyres index

This result of Laspeyres-Paasche bound is applicable to others

- As Balk (1998) summarizes, Laspeyres-Paasche bound is obtained for various index such as consumer price and quantity indices, and producer price and quantity indices.

- But there are no corresponding results for producer index..

What we did in this paper

- We show that Laspeyres and Paasche productivity indices constructs a bound for the Malmquist productivity index
- Isn't it obvious?
- No, because Laspeyres productivity index is Laspeyres output index divided by Laspeyres input index
- Unclear about how overestimations in numerator and denominator turn out to affect measurement of productivity

2. Index

Notation

- We measure productivity change from periods 0 to 1

- Output price and quantity

$$p = (p_1, \dots, p_M) \in \mathbb{R}^M_{++} \qquad y = (y_1, \dots, y_M) \in \mathbb{R}^M_{++}$$

- Input price and quantity

$$w = (w_1, \dots, w_N) \in \mathbb{R}^N_{++} \qquad x = (x_1, \dots, x_N) \in \mathbb{R}^N_{++}$$

Empirical index

- Laspeyres productivity index

$$LPI \equiv \left(\frac{p^0 \cdot y^1}{p^0 \cdot y^0}\right) \Big/ \left(\frac{w^0 \cdot x^1}{w^0 \cdot x^0}\right)$$

- Paasche productivity index

$$PPI \equiv \left(\frac{p^1 \cdot y^1}{p^1 \cdot y^0}\right) \Big/ \left(\frac{w^1 \cdot x^1}{w^1 \cdot x^0}\right)$$

Theoretical index

- Malmquist productivity index (period t is reference)

$$MPI \equiv \frac{D_i^t(y^0, x^0)}{D_i^t(y^1, x^1)}$$

- Laspeyres-Malmquist productivity index (period 0 is reference)

$$LMPI \equiv \frac{D_i^0(y^0, x^0)}{D_i^0(y^1, x^1)}$$

- Paasche-Malmquist productivity index (period 1 is reference)

$$PMPI \equiv \frac{D_i^1(y^0, x^0)}{D_i^1(y^1, x^1)}$$

3. Results

Proposition 1

- Suppose that a firm is engaged in profit maximizing behavior and its technology exhibits CRS, for both periods t=0 and 1. Then, the following two inequalities hold:
  - Laspeyres productivity index $\leq$ Laspeyres-Malmquist productivity index
  - Paasche-Malmquist productivity index $\leq$ Paasche productivity index

- Note: Unlike other empirical indices, Laspeyres index is the lower bound and Paasche index is the upper bound

- We rely on the theoretical bounds on the Malmquist productivity index derived by Balk (1993)

A simple illustrative example

- Output prices and quantities are fixed

$$p^0 = p^1 = \tilde{p} \qquad\qquad y^0 = y^1 = \tilde{y}$$

- Two inputs

$$w = (w_1, w_2) \qquad\qquad x = (x_1, x_2)$$

- Empirical index

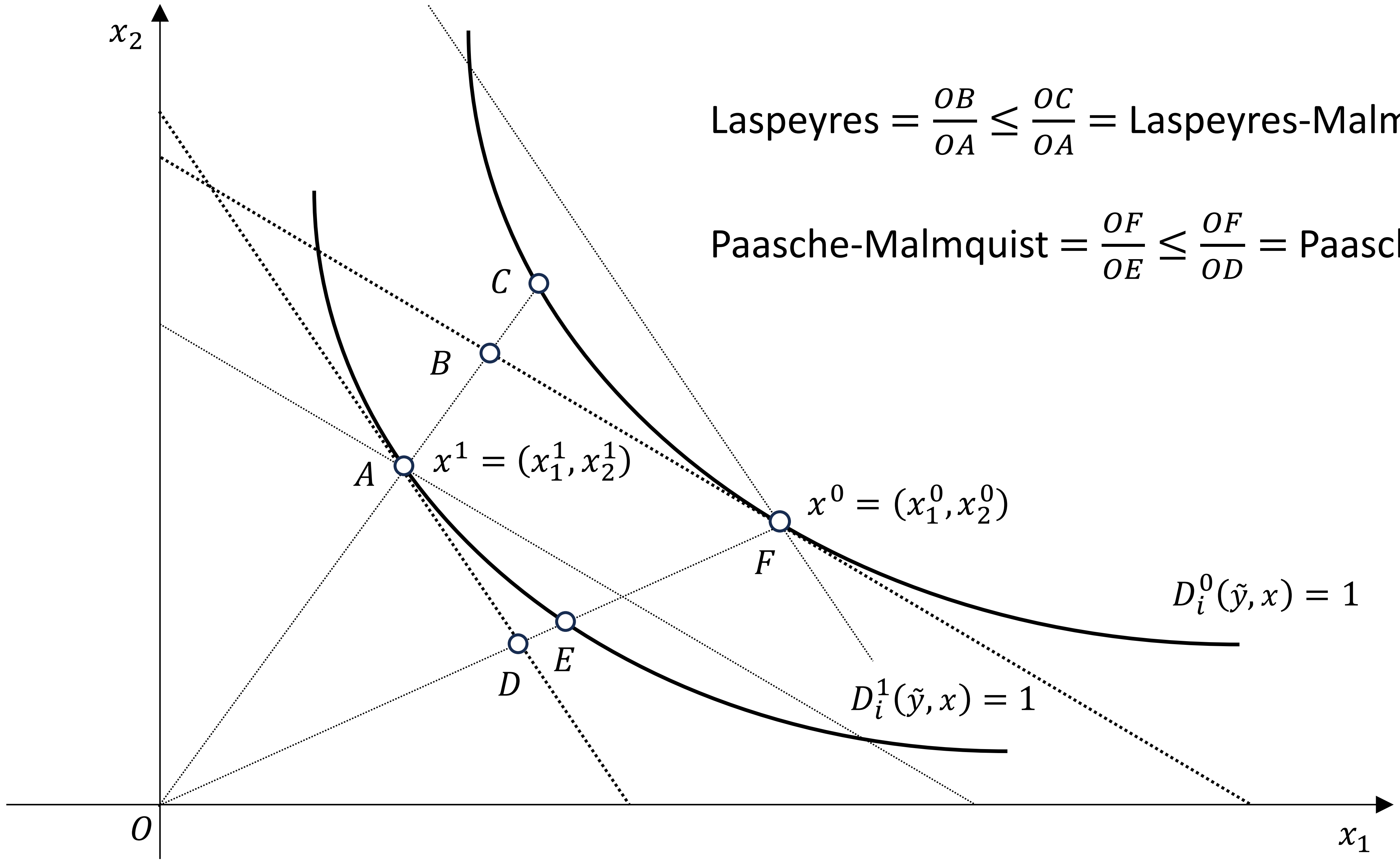
$$LPI \equiv \left(\frac{p^0 \cdot y^1}{p^0 \cdot y^0}\right) \Big/ \left(\frac{w^0 \cdot x^1}{w^0 \cdot x^0}\right) = \frac{w^0 \cdot x^0}{w^0 \cdot x^1} \qquad PPI \equiv \left(\frac{p^1 \cdot y^1}{p^1 \cdot y^0}\right) \Big/ \left(\frac{w^1 \cdot x^1}{w^1 \cdot x^0}\right) = \frac{w^1 \cdot x^0}{w^1 \cdot x^1}$$

- Theoretical index

$$LMPI \equiv \frac{D_i^0(y^0, x^0)}{D_i^0(y^1, x^1)} = \frac{1}{D_i^0(y^1, x^1)} \qquad PMPI \equiv \frac{D_i^1(y^0, x^0)}{D_i^1(y^1, x^1)} = D_i^1(y^0, x^0)$$

$$\text{Laspeyres} = \frac{OB}{OA} \leq \frac{OC}{OA} = \text{Laspeyres-Malmquist}$$

$$\text{Paasche-Malmquist} = \frac{OF}{OE} \leq \frac{OF}{OD} = \text{Paasche}$$



Proposition 2

- Suppose that a firm is engaged in the profit maximizing behavior and its technology exhibits CRS for both periods, t=0 and 1. When the technological change between periods 0 and 1 is **Hicks neutral**, the following relationship holds:
  - **Laspeyres productivity index** $\leq$ Laspeyres-Malmquist productivity index $=$ Paasche-Malmquist productivity index $\leq$ **Paasche productivity index**

- Note: Laspeyres productivity index $\leq$ Paasche productivity index does not imply Hicks neutral technological change, but the violation of this inequality imply non-Hicks neutral technological change (**Hicks neutrality test**)

4. Empirical application

US industry production data

- BEA-BLS Integrated industry-level production accounts

- Joint project of BLS and BEA which started from 2013
    - BLS works on measuring of productivity for US business sector
    - BEA works on constructing NIPA/industry GDP

- Comprehensive framework of industry productivity measurement consistent with NIPA/2008 SNA

- Price and quantity of input and output for 63 industries are available

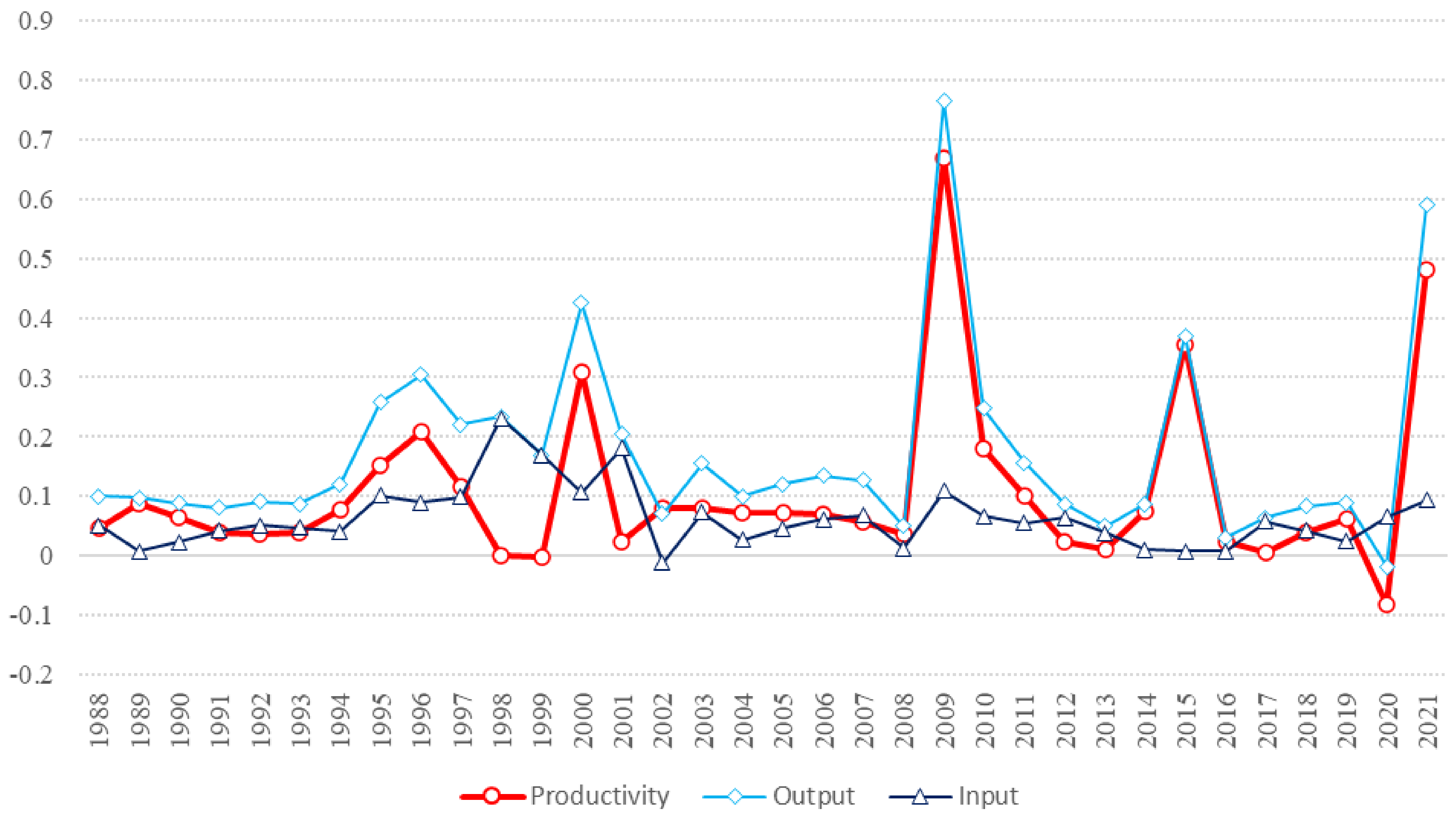
Aggregate productivity growth

- Each industry
  - 1 output: industry value added
  - 7 input: 5 capital + 2 labor

- Whole economy
  - $63 \times 1 = 63$ outputs
  - $63 \times 7 = 441$ inputs

- We can apply empirical index to prices and quantities of 63 industries to measure aggregate productivity growth

Difference between Laspeyres and Paasche index (% point)



What we found from US data

- Productivity index
  - Laspeyres is always larger than Paasche (except for 2020)
  - Frequent rejection of Hicks neutrality

- Output and input indices
  - Laspeyres is always larger than Paasche (except for 2002, 2020)
  - Almost no rejection of homotheticity

5. Conclusion

Summary

- We derive Laspeyres-Paasche bound for empirical productivity index
- As the case of price and quantity indices, this justify that the reasonable theoretical index lies between these two empirical indices
- If they are not far from each other, their average becomes a reasonable empirical index
- This is another justification of the Fisher productivity index taking a different route than Diewert (1992)

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